Abstract—This paper presents a proposal to modify power supply systems currently in use in ac-fed railways with neutral zones (NZs), in order to allow power-flow routing. The proposed system complements the existing infrastructure with additional power-electronic devices connected in parallel to both sides of the NZs, allowing control of power flow through adjacent electrical sections. The description and control of such a modified railway system is outlined in this paper. In addition, a mixed-integer programming optimization problem is formulated, which minimizes the investment and the operation costs, while ensuring the power supply to the train traffic. This optimization model is used to allow a systematic evaluation of the benefits of implementing such a railway smart grid system. Finally, a section of the high-speed line between Madrid and Barcelona is used as a case study, and the advantages of the proposed system are quantified in two different scenarios.

Index Terms—Controllability, efficiency, energy management, management, optimization, planning, power system, railway, smart grid.

I. INTRODUCTION

ELECTRIFIED railways are normally considered one of the most energy-efficient modes of transport, particularly over economically viable operating distances. One of the key factors for this higher efficiency is the interconnections of the trains via the catenary (or active rails in some cases), which allow trains to perform regenerative braking. In other words, a train equipped with a regenerative braking device, while undergoing deceleration due to braking, is able to act like a generator by efficiently feeding part or all of its kinetic energy, in the form of electrical power, to the traction electrical grid. For that reason, in the last two decades, research has focused on increasing the efficacy of the onboard regenerative braking systems by 1) enhancing the efficiency and the flexibility of the onboard electronic converters [1]–[3]; 2) optimizing the operation, for instance, by designing the train schedules or the driving to maximize the energy recovery [4]; and 3) enhancing the ability of the infrastructure to deal with excesses of power, for instance, by using energy-storage devices and reversible substations in dc systems.

The development of electrical smart grids is producing technologies that allow a rich interaction between the agents of the overall system (e.g., utilities, consumers, small generators, etc.) based on active management of the demand and the generation and active control of the electrical networks [5]–[7]. Although railway electrical grids are a particular case of electrical grid, some of their characteristics make them unique. First of all, the loads vary spatiotemporally because the locations of the trains and their power demands vary on almost on a continual basis. Furthermore, the number of loads is relatively small, although their load demand can be high. In addition, the loads are somewhat predictable because the nominal schedules of the trains are known in advance and a railway control center controls the movement of each train. Furthermore, from the point of view of the public grid, a fleet of moving trains can be considered to be a source of stored energy fed by the kinetic energy of the moving trains.

In three-phase power systems, power-flow routing has been traditionally performed by phase-shifting transformers, which are a special type of transformer, allowing to vary the phase shifting between the primary and the secondary side normally in a controlled way [8]. Since the late 1990s, the development of flexible alternating current transmission system devices has allowed different kinds of power-flow controls in ac transmission grids [9], normally relying on the notion of series/parallel compensation. More recently, similar concepts have been developed for distribution networks, particularly as the smart grid paradigm began to be adopted [10]–[12]. Finally, although direct conversion of the power (as opposed to series/parallel compensation schemes) is still problematic due to the large amount of power to be managed in distribution networks, it may be a feasible approach in the future.

This paper presents an enhancement for those railway power systems (RPSs) using segmented topologies. The proposed system allows an increased degree of controllability of the infrastructure, which enables, for instance, power routing.

In the field of railway electrification, RPSs are normally classified in two groups according to the characteristics of the voltage used in the power supply: low-frequency systems (which include dc, 16.7 Hz, 20 Hz, and 25 Hz) and industrial-frequency systems (50 Hz and 60 Hz). Although all these systems can use segmented topologies, it is in industrial-frequency systems where segmentation is commonly used [13].

This paper has five more sections. Section II describes the topology of the ac-fed RPSs with neutral zones (NZs) (often
referred to as industrial-frequency RPS). Section III describes the modification proposed in this paper and how it modifies the power distribution in a usual railway grid. Section IV proposes an optimization-based methodology to decide the dimensioning of the new elements to be installed. Its purpose is to evaluate the improvements that can be achieved with this technology and, therefore, its pertinence. This optimization methodology is then applied in Section V, using a case study based on a 550-km section of the Madrid–Barcelona high-speed line (HSL), and the results are analyzed. Finally, Section VI outlines the conclusions of this work.

II. POWER SUPPLY SYSTEMS USED IN RAILWAYS

Industrial-frequency RPSs are normally split into several feeding sections (FSs), each of which is fed from the three-phase public transmission or distribution grid (PTDG) through a single transformer located in a traction substation (TSS). The NZs are used to ensure electrical insulation between adjacent FSs.

Depending on the railways requirements, the FSs can be fed with the single-phase system with a neutral (referred to as 1×) or the unbalanced two-phase autotransformer (AT)-based system with a neutral (referred to as 2×), as illustrated in Figs. 1 and 2, respectively.

In the 2× system, although a two-phase system is set up, the loads are connected to only one phase (referred to as positive phase) and the neutral. The ATs are used to allow the flow of power from the other phase (referred to as the negative phase), which is unloaded [14].

In both Figs. 1 and 2, the symbols $U_{RS}$, $U_{ST}$, and $U_{TR}$ refer to the voltages in the PTDG used to feed each transformer. Normally, the phases are rotated to reduce the unbalances caused by the railway grid in the PTDG.

Fig. 1. Structure of a 1× RPS.

Fig. 2. Structure of a 2× RPS.

III. PROPOSED ENHANCEMENT TO POWER SUPPLY SYSTEMS

A. Description of the Modified System

The proposed modification consists in the addition of a power-transferring device (PTD) connected in parallel to both sides of each NZ (see Fig. 3). A PTD has to be able to transfer, from one FS to the other, the active and reactive power specified by a control system, which is referred to as an energy management system (EMS), for the positive phase, as well as for the negative phase (in AT-based systems). In Fig. 3, symbols $S_{pos,i}$ and $S_{neg,i}$ refer to the apparent power transferred by the positive and negative phases, respectively, at the side $i$, with $i = \{1, 2\}$.

It should be noted that, although the rated voltages are the same in all the FSs, there are phase shifts between adjacent FSs due to the phase selection when connecting the transformers to the PTDG. In addition, in 2× systems, a phase shift exists between positive and negative voltages.

Fig. 4 shows the architecture of the enhanced 1× RPS, in which, for the sake of clarity, two EMSs have been considered. Fig. 5 shows the architecture of the enhanced ac 2× RPS with, for the sake of clarity, also two EMSs. The acronym TSS-PTD refers to the PTDs located in the NZ of the TSS. The acronym NZ-PTD refers to the PTDs located in the other NZs.

Although the EMS architecture details are beyond the scope of this paper, NZs are normally operated by a substation (TSS or NZ-specific substation, depending on the cases), where reliable communication channels are available, allowing a centralized as well as a distributed-control system for the modified system. While the centralized architecture enables the control system to perform a global optimization of the operation, particularly in terms of energy-management efficiency, the distributed control yields enhanced redundancy for the railway systems.
B. Description of Operation of the Modified System

The power-balance expressions can be established for the general case (PTDs with star topology, with a higher number of terminals) as

$$\sum_{s=1}^{N_s} \sum_{p \in \{\text{pos}, \text{neg}\}} S_{p,s} = 0$$  \hspace{1cm} (1)$$

where $N_s$ is the number of sides of the PTDs.

For the two-side PTDs represented in Fig. 3, $N_s = 2$ and (1) becomes

$$S_{\text{pos},1} - S_{\text{pos},2} + S_{\text{neg},1} - S_{\text{neg},2} = 0.$$  \hspace{1cm} (2)$$

The power balance can be also expressed as a function of voltages and currents at the terminals of the PTDs, using

$$\sum_{s=1}^{N_s} \sum_{p \in \{\text{pos}, \text{neg}\}} (V_{p,s} \angle \theta_{p,s}) \cdot (I_{p,s} \angle \theta_{p,s} + \varphi_{p,s})^*$$  \hspace{1cm} (3)$$

where $V_{p,s}$ and $I_{p,s}$ are the voltage and the input current modules, respectively, in the terminal $(p,s)$ (side $s$ and phase $p$ of the PTD); $\theta_{p,s}$ is the angle of the voltage in the terminal $(p,s)$; $\varphi_{p,s}$ is the angle between the voltage and the current in the terminal $(p,s)$; symbol $^*$ represents the conjugate operand.

If the angle $\theta_{p,s}$ is taken as the reference in each terminal, then (3) can be expressed as

$$\sum_{s=1}^{N_s} \sum_{p \in \{\text{pos}, \text{neg}\}} (V_{p,s} \angle 0) \cdot (I_{p,s} \angle \varphi_{p,s})^*.$$  \hspace{1cm} (4)$$

Finally, if each phase is managed separately avoiding any power transfer between different phases, (4) becomes

$$\sum_{s=1}^{N_s} (V_{p,s} \angle 0) \cdot (I_{p,s} \angle \varphi_{p,s})^* = 0 \quad \text{for each phase } p.$$  \hspace{1cm} (5)$$

If the voltages on both sides of a PTD are assumed to have the same amplitude, then (5) reduces to

$$\sum_{s=1}^{N_s} I_{p,s} = 0 \quad \text{for each phase } p$$  \hspace{1cm} (6)$$

where $I_{p,s} = I_{p,s} \angle \varphi_{p,s}$.

Fig. 6 illustrates the way the currents in each of the phases are modified, by adding the PTDs to the infrastructure.
If $I_{\text{sup},k,t}$ and $I_{\text{sup},k+1,t}$ are expressed as a function of the currents through the PTDs, the expression (8) becomes

$$S_{\text{sup},k,k+1,t} = V_{\text{sup}} \sqrt{I_{\text{sup},k,t}^2 + I_{\text{sup},k+1,t}^2 + I_{\text{PTD},k+1,t}^2 - I_{\text{PTD},k,t}^2}.$$  \hspace{1cm} (9)

Two important remarks can be formulated based on (9). The power $S_{\text{sup},k,k+1,t}$ supplied by the substation to the sectors $k$ and $k+1$ does not depend on $I_{\text{PTD},k,t}$ (the current transferred by the TSS-PTD of the substation). Therefore, consequently, the real function of the TSS-PTDs has to be load balancing between the two transformers of the same substation.

### IV. Optimal Rating and Operation

In order to evaluate the advantages of the system, an operation strategy of the infrastructure has been considered, consisting in minimizing the total cost of the electricity supply. Here, an optimization model is proposed to determine the most efficient operation of the PTDs and the optimal investments in PTDs to be done. This model takes into account the following: 1) the train power consumptions that have been previously obtained with a rail traffic simulator [15] and 2) the electrification to be upgraded (electrical description of the substations and catenaries) [16].

As indicated in (10), the cost of electricity $C_{\text{elec}}$ is dependent on the usage of energy and the capacity of power allocated to a customer; that is

$$C_{\text{elec}} = C_{\text{ene}} \cdot E + C_{\text{pc}} \cdot P_{\text{max}}$$ \hspace{1cm} (10)

where $C_{\text{ene}}$ is the cost of the energy (in Euros per kilowatthrour), $E$ is the total energy consumption (in kilowathours), $C_{\text{pc}}$ is the cost of the allotted power capacity (in Euros per kilowatt), and $P_{\text{max}}$ is the allotted power capacity (in kilowatts). Depending on the case, $C_{\text{elec}}$ and $C_{\text{pc}}$ may depend on the specific hour of the day.

The optimization determines the value of the variables listed in Table I, in order to minimize the economic impact of installing and operating PTDs, which include not only the required investments but also the savings in the electricity bill due to the proposed enhancement.

The objective function to be minimized is as follows:

$$\text{INV} + \text{OC}.$$ \hspace{1cm} (11)

The investments include the cost of installing the PTDs in an existing infrastructure. Since voltage levels are known, the cost of each of the PTDs has been assumed to be proportional to the rated current of the PTDs, as follows:

$$\text{INV} = C_{\text{dev},1} \sum_{k \in \text{PTD}} I_{\text{rated},k}$$ \hspace{1cm} (12)

where $C_{\text{dev},1}$ is the cost per current unit of the PTD (in Euros, per ampere).

The current through the PTDs must be lower than its rated value to avoid a thermal destruction of the device. As the PTDs are bidirectional, the following constraint holds:

$$-I_{\text{rated},k} \leq I_{\text{PTD},k,t} \leq I_{\text{rated},k}$$ \hspace{1cm} (13)

The operating manageable costs include the costs of the losses incurred in the transformers and the catenary, as well as the cost of the allotted power capacity, as

$$\text{OC} = C_{\text{loss}C} + C_{\text{loss}X} + C_{\text{pow}}$$ \hspace{1cm} (14)

where $C_{\text{ene}} \cdot E = C_{\text{loss}C} + C_{\text{loss}X}$ and $C_{\text{pc}} \cdot P_{\text{max}} = C_{\text{pow}}$.

The cost of the losses in the transformer is as

$$C_{\text{loss}X} = C_{\text{ene}} R_{\text{e}} \delta_t \sum_{k \in \text{TR}} \sum_{t \in \text{T}} \left( I_{\sup,k+1,t}^2 + 2 I_{\sup,k,t} I_{\sup,k+1,t} - I_{\text{PTD},k,t}^2 \right) \hspace{1cm} (15)$$

where $\delta_t$ is the time step used in the traffic simulations, and the non-bold symbols $I_{xy}$ refer to the modules of the phasors $I_x$ for every index $x$ and $y$ (a power factor equal to one has been assumed).

The cost of the losses in the catenary is expressed by

$$C_{\text{loss}C} = C_{\text{ene}} \delta_t \sum_{t \in \text{T}} \sum_{j \in \text{CCS}_{k,t}} \left( R_{\ast}^j D_{j,t}(I_{j,t} - I_{\text{PTD},k,t})^2 \right)$$

$$+ C_{\text{ene}} \delta_t \sum_{t \in \text{T}} \sum_{j \in \text{CCS}_{k,t}} \left( R_{\ast}^j D_{j,t}(I_{j,t} - I_{\text{PTD},k-1,t})^2 \right) \hspace{1cm} (16)$$

where $\text{CCS}_{k,t}$ is the set of all the constant-current sections (CCSs) within the $k$th FS (see Fig. 7) at time step $t$, $I_{j,t}$ is the current in the $j$th CCS of a specific set $\text{CCS}_{k,t}$ at time step $t$, $D_{j,t}$ is the length of the $j$th CCS of $\text{CCS}_{k,t}$ at time step $t$, and $R_{\ast}^j$ is the resistance per length unit in the $k$th FS.
Equation (16) can be rewritten as

\[
C_{\text{loss}} = C_{\text{ene}} \delta_t \sum_t \sum_{k \in \text{CCS}} R'_k \\
\times \left( A_{k,t} + B_{k,t} I_{PTD,SOS,k,t} + C_k I^2_{PTD,k,t} \right) \\
+ C_{\text{ene}} \delta_t \sum_t \sum_{j \in \text{CCS}} R'_j D_{j,t} \\\n\times \left( A_{k,t} - B_{k,t} I_{PTD,k-1,t} + C_k I^2_{PTD,k-1,t} \right)
\]

(17)

where

\[
\begin{align*}
A_{k,t} &= \sum_{j \in \text{CCS}} D_{j,t} I^2_{j,t} \\
B_{k,t} &= 2 \sum_{j \in \text{CCS}} D_{j,t} I_{j,t} \\
C_k &= L_k
\end{align*}
\]

(18)

and \(L_k\) is the length of sector \(k\).

As the restrictions (17) and (15) are quadratic with the set of variables \(I_{PTD,k,t}\), only nonlinear solvers can be used to solve the optimization problem. Hence, the problem is transformed into a mixed-integer programming (MIP) problem by performing a piecewise linearization of the losses, in which the auxiliary variables described in Table II are considered.

To make the branch-and-bound process more efficient, special ordered sets type 2 (SOS2) have been used [17], [18].

The following additional restrictions are required for formulating the problem, in terms of the SOS2 variables

\[
\sum_s I_{PTD,SOS,k,t,s} = 1 \\
\sum_s I_{TR,SOS,k,t,s} = 1
\]

(19) (20)

Finally, the cost of the power capacity is given by

\[
C_{\text{pow}} = C_{\text{pc},I} \sum_{k \in \text{odd}} \max_t \left( I_{w/o,\sup,k,t} + I_{PTD,k+1,t} - I_{PTD,k-1,t} \right)
\]

(23)

where \(C_{\text{pc},I}\) is the cost of the allotted power capacity (in Euros, per ampere), assuming a given voltage in the power measuring point.

V. CASE STUDY

To evaluate the usefulness of the proposed system, a 549-km section of the HSL between Madrid and Barcelona is considered (from km 0 to km 549.153). In the study, the costs of the electrical power supply of the original and the optimized systems are compared.

Description of the case in Fig. 10 shows the simplified outline of the Madrid–Barcelona HSL. Except for the first 15 km, where the maximum speed is 250 km/h, trains can drive at 300 km/h for the rest of the studied section. There is an additional restriction in speed when arriving at Zaragoza (80 km/h at km 307) if the bypass is not taken.

Table III shows the sectors in this line section, the substations feeding them, their location, and the system (single-phase or two-phase). The location of the substations would also correspond to the location of the TSS-PTDs.

Table IV shows the location of the NZs of the line, which would also correspond to the location of NZ-PTDs.
Table V shows the electrical parameters of the railway line, including the characteristics of the power transformers and the series line parameters of the catenaries. To transform two-phase systems to single-phase system, the equivalent model described in [19] has been used.

Siemens S-103 trains have been considered, with trains traveling every 10 min in each direction. Fig. 11 summarizes the characteristics of these trains.

Based on the aforementioned data, the power consumption of the trains in both directions has been obtained. Figs. 12 and 13 show the power consumption and the speed of the trains in the Madrid–Barcelona and Barcelona–Madrid journeys, respectively, sampled every 5 s. A power factor of 1 has been assumed.

To simplify the evaluation of the performance of the system, the periodic traffic mesh, with trains every 10 min, has been considered to operate 9 hours per day, 365 days a year. The number of operating hours may seem a bit low, but the frequency corresponds to a peak period.

### A. Evaluation of the Enhanced System

In order to assess the advantages of the proposed system, its ability to control the power consumption of every substation and to reach an optimal operation is evaluated. However, the improvements are bounded by the rated currents of the PTDs, which depend on the investments: higher capacity PTDs may produce a more efficient operation, but are certainly more expensive items. As the proposed system is essentially a proof of concept, the prices of the PTDs are very difficult to estimate, particularly in the long term. For that reason, two scenarios have been studied.

- **Scenario A:** No cost has been considered for PTDs, i.e., $C_{dev,I} = 0 \text{ } €/A$, which leads to an optimal solution that only optimizes the operation, determining the rated currents of the PTDs that minimize the electricity costs. This case provides a good understanding of the best cost reduction that the system could reach if this technology would become massively adopted.
TABLE VI

<table>
<thead>
<tr>
<th>Power capacity</th>
<th>Cost of the electricity (averaged for &gt; 145 kV, 2012, Spain)</th>
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<tbody>
<tr>
<td>23.92 €/kW/year</td>
<td>598 €/A/year at 25 kV</td>
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<tr>
<td>0.07 €/kWh</td>
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- Scenario B: The cost of PTDs has been assumed to be 410 €/kVA. In addition, 10% of this cost will be considered each year to be balanced with the electricity bill reductions, which makes $C_{dev,t} = 1025 \text{ } €/A$ (at 25 kV).

In this case, the optimization will find a tradeoff between investments and energy savings, which gives a reference of the benefits of the proposed system for reducing the electricity costs.

Figs. 14 and 15 compare the power supplied by every substation in scenarios A and B with the actual electrification,
Fig. 14. Current supplied by the TSS for each instant. Reference versus Scenario A.
Fig. 15. Current supplied by TSS for each instant. Reference versus Scenario B.
which is used as the reference. With the only exception of the substations “Montagut” and “L’Espluga” in scenario A, the maximum power peaks are significantly reduced. With the proposed system, all the power supply of the substation “La Gornal” is even effectively assumed by the other substations.

Table VIII compares the rated values of the PTDs in both scenarios. In scenario A, as the cost of the PTDs is not considered in the objective function, the rated current of TSS-PTDs takes a nonzero value, in order to minimize losses in the transformers (as discussed previously, the function of the TSS-PTDs is mainly to balance the load supplied by the two transformers of the substation). In scenario B, the cost of the PTDs is largely higher compared to the cost of the transformer losses that can be saved by load balancing. Therefore, the optimization leads to not install TSS-PTDs at all.

Table VIII summarizes the enhancements due to adopting the proposed system in the analyzed line. The results obtained for the reference case shows that 94% of the manageable cost of the electricity corresponds to the power capacity term, while the losses represent 4% (in the catenary) and 2% (in the transformers). The manageable cost of the electricity includes the cost of the power capacity and the cost of the losses in the catenary and the transformers, but excludes the cost of the energy consumed by trains (which is assumed not to be controlled by the infrastructure).

In scenario A, where PTDs can be rated to obtain the best improvements in the system regardless of their cost, the enhanced system would be able to cut down 32% of the power capacity costs and, very similarly, the losses. In this specific case, this would be an upper bound of the enhancement that the system could reach.

In scenario B, where real prices and a chargeoff of ten years have been considered, improvements are lower than in scenario A. However, a reduction of 20% in the manageable costs is reached, mainly due to the savings in the power capacity term (−21%). To achieve this, the system is able to route the electrical power from different substations to the sectors where it is required. In exchange, the currents have to cross longer distances, and the electrical losses rise up (+5%). The losses in the transformer are, however, reduced (−4%).

VI. Conclusion

This paper has presented a system to improve the ac railway power supply systems, which have NZs. The system allows an improved degree of controllability of the infrastructure, which allows for instance power routing in traction electrical grids. The proposed system could be an important milestone in the railway smart grid roadmap.

The proposed system would be able to route the electrical power routing, making possible new ways of operation of railway systems more reliable and cost efficient.

As an example of such intelligent operation of the RPS, a strategy focused on the minimization of the manageable costs of the power supply (including costs of the power capacity and losses) has been considered in a study case, which corresponds to a 550-km long section of the Madrid–Barcelona HSL. The system would be able to reduce up to 31% of these manageable costs of the electricity.
REFERENCES


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