# Self-Triggered Distributed Control and Non-linear Dynamic Optimization of a Microgrid

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Self-Triggered Distributed Control and Non-linear Dynamic Optimization of a Microgrid

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Abstract—In this paper dynamics of a power system consisting of distributed generators in a microgrid is modeled as a nonlinear system. The specific objectives are to optimize nonlinear economic-emission dispatch and control the dynamics of power sharing of distributed generators in a microgrid. For efficient coordination of distributed generators, a multi-agent system with shared communication framework and employing self-triggered data sampler is considered. Specifically, agents sense the state space and share the information with neighboring agents using self-triggered aperiodic data sampler. The system is tested for load as well as source side transients. The performance evaluation results show that distributed optimized control with proposed self-triggered strategy significantly reduces the bandwidth requirement. In addition, the dynamic performance of self-triggered data sampler is compared with that of periodic data sampler. Numerical results show that approximately thirty times less data transmission is required during steady state while data transmission requirement during transients reduces six to seven times, leading to significant reduction in the demanded communication bandwidth.

Index Terms—Microgrid, distributed generation, multi-agent system, self-trigger sampler.

I. INTRODUCTION

With the emergence of smart grid, consumers are using enhanced levels of information and control to reduce the length of outages, improve system efficiency, better resource utilization and reduced gaseous emissions. To meet the growing energy demand, distributed energy resources (DERs) such as Photovoltaic (PV) system, wind turbines, micro-gas turbines, fuel cells etc. have now became integral part of small scale community microgrids (MGs) and their penetration into modern power system is on rise. The presence of these DERs in MGs has given birth to distributed generation. The integration of DERs in MGs, however, creates new challenges such as resource scheduling, dynamic economic dispatch, power quality, etc. This demands for efficient communication infrastructure enabling distributed control and monitoring of DERs in real time.

Integration of each DER in an MG can be modeled as an agent in the context of a multi-agent system (MAS).

Traditionally, control and monitoring of these DERs is carried out by using sensors which share information periodically. In a sampled information system, sampling rate is selected to meet worst case system error constraints. However, the worst case sampling rate requirement for transients is not essential during steady state. Rather sampling rate can be reduced significantly, during steady state, without compromising the performance. Due to limited power network bandwidth of the microgrid, resources can not be utilized effectively and therefore, an efficient communication infrastructure is desired for a scalable system [1], [2]. Fortunately, a need based or event based information exchange is a solution to this problem which results in aperiodic information transfer. This has clear benefits in contrast to periodic information exchanges. The aperiodic information exchange is a paradigm shift from conventional periodic data sampling. Aperiodic information exchange can be implemented using two methods reported in the literature, namely event-triggered based data exchange or self-triggered approach [3], [4]. These two methods have been proposed for different applications including network control systems [5], distributed power sharing among microgrids, formation control of multi-robot systems [6] and wireless sensor networks [7] to name a few.

Optimal power flow in microgrids using event-triggered distributed optimization is proposed in [8] where each subsystem transmits its local state to its neighbors in case an event or abnormal situation occurs and error signal exceeds a certain state dependent threshold. Aperiodic event-triggered control of multi-agent systems has been proposed in [9]. A distributed load sharing approach for inverter based microgrid with aperiodic information exchange is proposed in [10] for the coordination of multiple intermittent DGs. Specifically multi-agent system based discovery algorithm has been used to discover global microgrid information. New control and aperiodic event-triggered communication strategies for distributed energy management of both generators and loads is proposed in [11].

Event-triggered data communication interface generates a response when the system variable has violated certain threshold from a nominal value. Authors in [12] have proposed decentralized event-triggered control of nonlinear systems. The emphasis is on sensor data transmission to a central controller at the arrival of certain events. Many efforts are in progress, where nonlinear controllers are being proposed with event-trigger communication for different application scenarios, including active and reactive power sharing in microgrids [13], cost optimization in network control systems [14], distributed dynamic consensus [15] and for economic dispatch of DC
microgrids [16] and adaptive event-triggering for secondary control of DC microgrids [17]. In contrast to event-triggered which is reactive in nature, self-triggered communication strategy is highly a proactive approach evaluating instances ahead of time at which information can be mutually shared. For event-triggered communication, continuous state monitoring is required. However, for self-triggered communication strategy agent states are only observed at the triggering instances [9].

Due to the above mentioned facts, a self-triggered mechanism for data transfer among the agents in a multi-agent based microgrid has been selected in this research work to perform power sharing control among the DGs. Specifically, we have introduced a non-linear dynamic model of microgrid with economic-emission dispatch and supply-demand balance. Performance of multi-agents as reported in [18] results in average consensus of DGs having equal power rating. In a real and practical scenario, economic dispatch of unequal load sharing among the DGs is possible and usually is the case. The key contributions of this paper include: (a) a self-triggered communication based strategy for real time non-linear economic-emission dispatch and power sharing among distributed generators, and (b) an augmented Lagrangian based optimized PID controller that provides tight generator as well as load side transient control of the DGs.

Rest of the paper is organized as follows. A system model is proposed for a microgrid with multiple distributed generation having non-linear economic-dispatch with PV integration in Section II. Multi-agent communication framework along with distributed optimization based problem formulation is also outlined in this section. Details of implementing a self-triggered data sampler for economic-dispatch dispatch is presented in Section III. Performance evaluation results for the proposed approach are provided in Section IV followed by conclusions in Section V.

II. SYSTEM ARCHITECTURE FOR MICRORGRID

System architecture of the proposed microgrid is shown in Fig. 1 consisting of several conventional thermal generators and renewable PV sources connected to a common bus. These generating units supply power to the load. In an islanded mode of microgrid operation, whenever there is a power fluctuation due to either variation in power generated by distributed generators or variation in load, there is a transient resulting in a degraded power dynamics. The microgrid under consideration supplies power to few MW load so these transients can not be ignored. An efficient control mechanism is highly desirable which can correct these transients and improve the dynamic performance of the system.

A. Communication Network Architecture

The system model for communicating DGs of different types in the microgrid is shown in Fig. 1. It is assumed that renewable energy PV sources are present as distributed generators and are a source of transients. Each DG is modeled as an agent in the multi-agent architecture of the microgrid and communicates with its neighbors through a wired or wireless communication interface. Network of connected DGs can be modeled as a spectral graph and all the properties of graph associated with characteristic polynomials, eigenvalues, eigenvectors of matrices such as its adjacency matrix or Laplacian matrix can be used for distributed implementation. We select \( N \) DGs which are connected by their bidirectional communication links. The whole network of DGs can be considered a graph \( G = \{ N, W \} \). \( W \) represents the communication links between the DGs which can communicate. The Laplacian matrix \( M \) is defined as \( M = D - A \) where \( D = D(G) \) is degree matrix and \( A = A(G) \) is the adjacency matrix [19]. The communication delays in transmitting data between a pair of DGs is considered much lower compared to time required by the controller to update its output.

B. Power Generation Model for Microgrid

We formulate a non-linear dynamic model based on economic-emission dispatch (EED) of a microgrid with PV integration. The EED problem has two components, namely economic dispatch and emission dispatch. Both components are modeled as quadratic functions. The optimization objective function therefore, consists of minimizing generation cost of DGs for economic dispatch and reducing the cost of pollutant emissions in emission dispatch. The two cost functions can be added together because of their quadratic nature. \( \text{CO}_2 \) is the most harmful pollutant reported in literature besides \( \text{NO}_x \) and \( \text{SO}_x \). Generation cost in an economic dispatch \( J_i(p_i) \) is defined as

\[
\sum_i J_i(p_i) = \sum_i \alpha_i p_i^2 + \beta_i p_i + \chi_i, \quad \forall i, \tag{1}
\]

while pollutant emission cost, \( E_i(p_i) \) is given by

\[
\sum_i E_i(p_i) = \sum_i a_i p_i^2 + b_i p_i + c_i + \zeta \exp(\lambda p_i), \quad \forall i. \tag{2}
\]

In (1), \( p_i \in \mathbf{p}, \mathbf{p} \in \mathbb{R}^N \) represents power generation vector, while \( \alpha_i, \beta_i \) and \( \chi_i \) are coefficients representing the generation cost of the \( i^{th} \) generator. In (2), \( a, b, c, \zeta \) and \( \lambda \) are pollutant...
emission cost coefficients whose values are supplied in Table I. Finally, economic-emission cost function, \( F_i(p_i) \) is obtained by adding (1) and (2), as given below
\[
\sum_i F_i(p_i) = \sum_i A_i p_i^2 + B_i p_i + C_i + \zeta \exp(\lambda p_i), \; \forall i, \quad (3)
\]
where \( A_i = \alpha_i + a_i \), \( B_i = \beta_i + b_i \) and \( C_i = \chi_i + c_i \).

Presence of PV power as a renewable energy source in the microgrid will affect the power sharing of DGs. This fact is attributed to the intermittent nature of PV power resulting in power fluctuations on the main bus. To account for uncertain power fluctuations, a PV cost function is added in the objective function. The running costs of photo-voltaic units are assumed minimal and have not been taken into consideration. The cost function \( T(p_{PV}) \) for lumped PV power can be mathematically expressed as
\[
T(p_{PV}) = z_1 p_{PV} + \varepsilon z_2 \exp(z_2 - p_{PV}) \quad (4)
\]
where \( z_1 = 0.9 \), \( z_2 = 1.7 \), \( \varepsilon_z = 108 \) and \( p_{PV} = 1MW \) as maximum PV power used for simulations. \( T(p_{PV}) \) has two components: direct operating cost being the first term in (4) while the second term in (4) corresponds to the price on curtailment of PV power generation [21].

An optimization problem can now be formulated having considered all the above mentioned components as follows,
\[
\min \sum_i F_i(p_i) + T(p_{PV})
\]

Subject to
\[
\begin{align*}
\sum_i p_i + p_{PV} &= L_d + L_{d EV}, \\
M p + \delta &\geq 0, \quad p_{i min} \leq p_i \leq p_{i map}.
\end{align*}
\]

In (5), \( M \in \mathbb{R}^{N \times N} \) and \( \delta \in \mathbb{R}^N \), while \( L_d \) denotes the load demand. The first constraint in (5) is the power generation and demand balance, whereas second constraint is called the relaxed consensus constraint. The desired relaxation is obtained by varying relaxation coefficient \( \delta \). Average consensus is obtained when \( M p = 0 \). By letting \( \delta \geq 0 \), greater flexibility is achieved in further minimizing the cost of generation. The last group of constraints in (5) are responsible to keep all generator’s power levels between their minimum and maximum limits. In (5), \( L_{d EV} \) is a distributed charging load of electric vehicles. By varying these distributed loads, microgrid transients can be further studied.

C. Distributed Control of Microgrid

The microgrid has fossil fuel based generators and DERs. Using Augmented Lagrange multiplier approach [19] an optimization problem can now be formulated. The augmented Lagrangian function \( L_a \) is given by
\[
L_a(p, \lambda, \rho, \sigma, \bar{p}) = \sum_i \omega_i (F_i(p_i) + T(p_{PV})) + \sum_i k_p \frac{1}{2} (L_d + L_{d EV} - \sum_i p_i - p_{PV})^2 \nonumber
\]
\[
+ k_i \left( \lambda \left( L_d + L_{d EV} - \sum_i p_i - p_{PV} \right) + \Theta_i \sum_i (M p + \delta) + \sum_i \rho(p_{min} - p_i) \right. \nonumber
\]
\[
+ \left. \sum_i \sigma(p_i - p_{map}) \right) + \sum_i k_d \frac{1}{2} (p_i - \bar{p}_i)^2 \quad (6)
\]

and the corresponding dynamic equations for microgrid are obtained using augmented Lagrangian (6) as follows.
\[
\begin{align*}
\dot{p}_i &= k_p (F_i(p_i) + u_t), \quad \forall i \\
\dot{p}_{PV} &= k_p T(p_{PV}) \\
\dot{\lambda} &= k_\lambda \left( L_d + L_{d EV} - \sum_i p_i - p_{PV} \right)^+, \quad \forall i \\
\dot{\Theta_i} &= k_{\Theta_i} \left\{ \sum_i (M p + \delta) \right\}^+, \quad \forall i
\end{align*}
\]

For enhanced dynamic performance, an optimized solution with control gains \( k_p, k_\lambda \) and \( k_{\Theta_i} \) are included in the control law \( u_t \) that can be derived from above expression and is finally given by:
\[
u_t = -k_p^{(i)} (\lambda - \Theta_i M_i) + k_d^{(i)} (p_i - \bar{p}_i) - k_p^{(i)} \psi(\lambda), \quad \forall i\]  
(8)

III. SELF-TRIGGERED DATA SAMPLING FOR NON-LINEAR SYSTEM DYNAMICS

In this section mathematical formulation for self-triggered data sampler for microgrid non-linear system dynamics is provided.

A. Notations and Nomenclature

Let \( \mathbb{R}^+ \) be non-negative real numbers and \( p \in \mathbb{R}^+ \). Define Euclidean norm of signal \( p \) as \( \|p\| \) and \( \|p\|_{L_{\infty, k}} := \sup_{t \in \mathbb{Z}_k} \|p(t)\| \). If a function \( f(t, p) \) is continuous in \( t \) and \( p \) then its solution \( p(t) \) is continuously differentiable. Given the state equation of physical system,
\[
\dot{p} = f(t, p), \quad p_0 = p(t_0)
\]
the function \( f(t, p) \) must satisfy the following inequality Lipschitz condition,
\[
\|f(t, p) - f(t, y)\| \leq L \|p - y\|, \quad (10)
\]
for all \( (t, p) \) and all \( (t, y) \) in the neighborhood of \( (t_0, p_0) \) where \( L \) is Lipschitz constant. \( f \) is Lipschitz continuous in \( p \) but piecewise continuous in \( t \). Another important aspect is that the solutions of (9) are expected to be unique, bounded and
have finite values. This requires that \(\|p(t)\| \leq \tau\) for all \(t > t_0 + T\) where \(\tau\) is some finite value constant. The value of \(\tau\) is referred to as ultimate bound.

B. Self-Triggering Condition

In this section, self-triggered sampler for nonlinear system will be evaluated. We have determined, dynamics of power system for economic-emission dispatch in (7). We can represent (7) by a general nonlinear system of the form [22].

\[
p = f(p, k(p)) = f(p, u)
\]

(11)

where \(p \in K_p \subset R^n\) is a state vector and \(u \in K_u \subset R^p\) is an input vector and \(K_p, K_u\) are domains containing the origin. Assume that there exists a state feedback control law \(k : K_p \rightarrow K_u\) such that closed loop system given by (11) is asymptotically stable in the origin. \(f(p, k(p)) \subset C^1(K_p \times K_u)\) must have Lipschitz continuous derivatives for this condition to hold. We need continuous measurements of the state \(p(t)\) to execute the control law \(u(t) = k(p(t))\). However, if state data is sampled at discrete time instants \(t_k, k \geq 0\) the control law is \(u(t_k) = k(p(t_k))\) for \(t \in [t_k, t_{k+1})\) and system dynamic equations are modified to

\[
p = f(p, k(p_k))
\]

(12)

The problem now is to design a self-triggered sampler such that solution of the system given in (12) is bounded. The set of sampling time instants \(v = \{t_k\}, k \geq 0\) need to be determined to ensure asymptotic stability of the system. How to design such time instants are shown in [22]. The self-triggered sampling problem and associated stability of the system can now be defined. Given a nonlinear system such as (12), determine a function \(\beta_v : K_p \rightarrow [\beta_{min}, \infty)\), with \(\beta_{min} \geq 0\) and a minimum sampling time \(t_{k+1}, k \geq 0\) such that

\[
t_{k+1} = t_k + \beta_v(p(t_k))
\]

(13)

then origin of the closed loop system of (11) with state feedback control \(u = k(p_k)\) is asymptotically stable over the set \(K_k\). If self-triggered sampler is designed properly, it will also ensure \(t_{k+1} - t_k \geq 0\) for \(k\). The control signal is updated whenever it hits a threshold \(\gamma\) such that the Euclidean norm of \(E(t)\) is

\[
\|E(t)\| := \|f(p(t), k(p_k)) - f(p, k(p(t))\| \leq \gamma, \gamma > 0.
\]

(14)

This expression in (14) is called triggering condition or error function. Whenever triggering condition is deviated, it calls for another updated sampling and this process is repeated till the error is zero.

Let \(u(v) = k(p(v))\) and \(E(v) = f(p(t), u(t_k)) - f(p(t), u(v))\). Taking the derivative w.r.t. \(v\), it employs that

\[
\left\{ \frac{d}{dv} E(v) = -\frac{d}{dv} f(p(t), u(v)) := \varphi(p(t), u(v)), E(v_k) = 0 \right\}
\]

Integration on both sides results in,

\[
E(v) = \int_{v_k}^{v} \varphi(p(t), u(\sigma)) d\sigma
\]

\[
= \int_{v_k}^{v} \varphi(p(t), u(\sigma)) - \varphi(p(t), u_k) d\sigma + \int_{v_k}^{v} \varphi(p(t), u_k) d\sigma
\]

(15)

Taking norms on both sides of (15),

\[
\|E(v)\| \leq \int_{v_k}^{v} \|\varphi(p(t), u(\sigma)) - \varphi(p(t), u_k)\| d\sigma + \int_{v_k}^{v} \|\varphi(p(t), u_k)\| d\sigma
\]

(16)

Utilizing Leibniz Theorem,

\[
\frac{d}{dv} E(v) = \varphi(p(t), u_k) \leq \|\varphi(p(t), u_k)\| E(v_k) \leq \|\varphi(p(t), u_k)\| E(v_k) \leq \|\varphi(p(t), u_k)\| E(v_k) \leq \|\varphi(p(t), u_k)\| E(v_k)
\]

(17)

and then

\[
E(v) = \varphi(p(t), u_k) = \frac{E(p_k, t, t_k)}{L_{\varphi(p,u)}}
\]

(18)

By replacing \(p\) with \(p^*\), it follows that error function is upper bounded by

\[
\|E(v)\| \leq \frac{\|\varphi(p^*(t), u_k)\|}{L_{\varphi(p,u)}} (e^{L_{\varphi(p,u)}(v-v_k)} - 1) := E(p_k, t, t_k), \forall p.
\]

(19)

where \(L_{\varphi(p,u)}\) is the Lipschitz constant. It is important to note that \(L_{\varphi(p,u)}\) and \(p^*\) are needed at each sampling instant for proposed self-triggered sampler. Therefore following proposition can be used to calculate \(L_{\varphi(p,u)}\) [23].

\[
L_{\varphi(p,u)} = \frac{\varphi(x) - \varphi(y)}{y - x}
\]

(20)

and optimal values for \(p^*\) are calculated

\[
p^* = \max_{y \in K_p} \|\varphi(y, k(p_k))\|
\]

(21)

Finally, self-triggered data sampling condition can be designed that ensures stability of the system and providing the desired performance. From (18), when \(E(p_k, t, t_k) = \gamma\), equals the error threshold, (18) becomes

\[
\beta_v(p(t_k)) = \frac{1}{L_{\varphi(u)}} \log \left(1 + \gamma \frac{L_{\varphi(p,u)}}{\|p^*(t_k)\|} \right)
\]

(22)

Combining (13) and (22), the self-triggered sampler is given by,

\[
t_{k+1} = t_k + \frac{1}{L_{\varphi(u)}} \log \left(1 + \gamma \frac{L_{\varphi(p,u)}}{\|p^*(t_k)\|} \right)
\]

(23)
The architecture of self-triggered control mechanism is shown in Fig. 2. Event detector continuously monitors system states, a sampling event is triggered when the sampler receives a sample message. A self-triggered sampling event is triggered when $t = t_k + 1$. This time stamp is provided by self-triggered scheduler. The state and control law are represented by $p(t)$ and $u(t)$, respectively. The Lyapunov stability for the proposed system is given in Appendix A.

IV. PERFORMANCE EVALUATION RESULTS

For optimal power flow, seven DGs are considered with their connectivity graph similar to as described in [24]. A case of multi-agent system connected through a communication interface for seven DGs is illustrated in Fig. 1, results in the following Laplacian matrix, $M$ for this graph and is given by

$$M = \begin{bmatrix}
2 & -1 & -1 & 0 & 0 & 0 & 0 \\
-1 & 3 & -1 & -1 & 0 & 0 & 0 \\
-1 & -1 & 2 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 4 & -1 & -1 & -1 \\
0 & 0 & 0 & -1 & 2 & -1 & 0 \\
0 & 0 & 0 & -1 & -1 & 3 & -1 \\
0 & 0 & 0 & -1 & 0 & -1 & 2
\end{bmatrix}. \quad (24)$$

The power generation capacities along with generation cost parameters for the seven generators are tabulated in Table I [25]. Maximum power demand from load side is taken as 10 MW in this study.

A. Dynamic Performance Comparison of Periodic and Self-triggered Communication

To check dynamic performance and verify plug-and-play capabilities, transients are introduced both on load and generator sides. Fig. 3 depicts the dynamics of DG powers for this scenario. As shown in Fig. 3, DG1 and DG3 in the microgrid are intentionally disconnected between 5 – 10 and 25 – 30 sec time instants. DGs join back the microgrid after some time. Load transient is introduced by increasing load between 18 – 23 sec time instants. Dynamic performance is compared for periodic sampling with that of self-trigger sampling, both employing optimized PID control. It can be observed that self-trigger sampling has comparable dynamic performance as far as transient response is concerned. The results show that using self-trigger strategy does not harm the dynamic performance of the microgrid.

B. Bandwidth Efficiency and Effect of Transients

By setting $\gamma = 0.031$ and $\delta = 1$, the upper bound on time difference, $t(k+1) - t(k)$, the inter-sample time during steady state has increased to 0.06 sec in self-triggered sampler as compared to periodic data sampling implementation in which inter-sample time is fixed to 0.002 sec. This is a huge saving in terms of data sampling, approximately thirty times less sampling during steady state operation. The inter sample time is decreased when there is a transient in the system, as shown
in Fig. 4, resulting in higher sampling rate. Inter-sample time is reduced from 0.07 sec to about 0.01 sec during transients resulting in seven times reduced communication bandwidth.

Further in a similar scenario, both load and PV power transients are introduced and their effect is studied on inter-sample time \( t_{k+1} - t_k \) for self-triggered sampling.

C. Performance of Self-triggered Sampler for Distributed Electric Vehicle Load

To study the performance of self-triggered sampler for a distributed charging load of Nissan Leaf electric vehicle, an EV charging/discharging curve is used. Measured data for such a vehicle is taken from [26] having 50kW maximum charging capacity. Considering a fleet of 100 such vehicles with a total of 5MW charging, as shown in Fig. 6, the performance of self-triggered sampler can be easily visualized from the graph. The inter-sampling time varies continuously following the charging and discharging curve. Each incremental charging and discharging results in a transient and self-triggered sampler follows it closely.

D. Variation of Sampling Rate with Parameter \( \gamma \)

In this case the performance of self-triggered optimized PID is studied. \( \gamma \), which is an error threshold defined in (14), is taken as parameter and variation of sampling rate (no of samples per given time) is studied. The results are shown in Fig. 7. Sampling rate decreases in a small range of \( \gamma \) values i.e. from 0.01 to 0.08 in steps of 0.005. Tuning gains \( K_p \), \( K_i \) and \( K_d \) are provided in Table I. From parametric graph between sampling rate and \( \gamma \) shown in Fig. 7, any sampling rate can be fixed for a given error threshold for desired system performance.

E. Effect of \( \delta \) Variation on Sampling Rate

\( \delta \) is a parameter used as a relaxed consensus constraint as given in (5). Its effect on sampling rate is shown in Fig. 8. From parametric graph between sampling rate and \( \delta \), any sampling rate can be obtained for desired system performance.

V. Conclusions

Non-linear economic-emission dispatch problem and optimized control for distributed generation is considered for a typical microgrid in this work. An optimized PID type power

<table>
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<th>Parameter</th>
<th>DG1</th>
<th>DG2</th>
<th>DG3</th>
<th>DG4</th>
<th>DG5</th>
<th>DG6</th>
<th>DG7</th>
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<td>( \alpha \times 10^{-3} )</td>
<td>1</td>
<td>1.5</td>
<td>2.5</td>
<td>2</td>
<td>1.5</td>
<td>2.5</td>
<td>2</td>
</tr>
<tr>
<td>( \beta \times 10^{-3} )</td>
<td>5</td>
<td>2</td>
<td>2.5</td>
<td>4</td>
<td>2</td>
<td>2.5</td>
<td>4</td>
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<tr>
<td>( \gamma )</td>
<td>0.10</td>
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<td>0.09</td>
<td>0.075</td>
<td>0.15</td>
<td>0.09</td>
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<td>( \alpha \times 10^{-3} )</td>
<td>4.091</td>
<td>2.543</td>
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<td>5.426</td>
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<tr>
<td>( b \times 10^{-3} )</td>
<td>-5.554</td>
<td>-6.047</td>
<td>-5.094</td>
<td>-3.550</td>
<td>-6.047</td>
<td>-5.094</td>
<td>-3.550</td>
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<tr>
<td>( c \times 10^{-3} )</td>
<td>6.490</td>
<td>5.638</td>
<td>4.586</td>
<td>3.380</td>
<td>5.638</td>
<td>4.586</td>
<td>3.380</td>
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<tr>
<td>( K_p \times 30 )</td>
<td>0.32</td>
<td>0.35</td>
<td>0.32</td>
<td>0.25</td>
<td>0.30</td>
<td>0.25</td>
<td>0.28</td>
</tr>
<tr>
<td>( K_i \times 30 )</td>
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<td>0.50</td>
<td>0.60</td>
<td>0.36</td>
<td>0.45</td>
<td>0.54</td>
<td>0.62</td>
</tr>
<tr>
<td>( K_d \times 0.5 )</td>
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<td>30</td>
<td>30</td>
<td>20</td>
<td>25</td>
<td>30</td>
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</tbody>
</table>

Max Power (MW) | 2   | 2   | 2   | 2   | 2   | 2   | 2   |
Load Demand (MW) | 1 – 10 |
EV Load (kW) | 500 |

Fig. 4. Inter-sample time \( t_{k+1} - t_k \) for self-triggered sampling.

Fig. 5. DG powers with load and PV power transients (top) and Inter-sample Time \( \Delta t = t_{k+1} - t_k \) with transients (bottom).

Fig. 7. Sampling rate decreases in a small range of \( \gamma \) values i.e. from 0.01 to 0.08 in steps of 0.005. Tuning gains \( K_p \), \( K_i \) and \( K_d \) are provided in Table I. From parametric graph between sampling rate and \( \gamma \) shown in Fig. 7, any sampling rate can be fixed for a given error threshold for desired system performance.

Fig. 8. Effect of \( \delta \) Variation on Sampling Rate
width reduction and improved transient dynamic performance. achieving significant gains in terms of communication band-

network scenarios having many interconnected microgrids thus proposed solution can be extended for more complex power
desired sampling rate for desired system performance. The requirements. Parametric graphs are extracted for selecting significantly reduces the system’s communication bandwidth tested on the microgrid. The proposed self-triggered strategy theory. Self-triggered data sampling conditions are applied and based optimization technique is applied to design the con-
PV power integration. Specifically, augmented Lagrangian control is tested for a microgrid having thermal generators with

Fig. 7. Sampling rate variation with parameter $\gamma$: self-triggered optimized PID control.

APPENDIX A

LYAPUNOV STABILITY ANALYSIS

In the Lyapunov approach a function $Z(p, t)$ is considered a Lyapunov candidate function if

$$
\omega_i \|p\| \leq Z(p, t) \leq \omega_j \|p\|, \\
Z(p, t) = 0, \text{ for } p = 0
$$

(25)

with $\omega_i$ and $\omega_j$ being $K$-class functions which are defined in $\mathbb{R}^+$, monotone non-decreasing, continuous, null at the origin and indefinitely increasing with its argument. The dynamic system is stable if the derivative of candidate Lyapunov function w.r.t. time is negative, representing an energy dissipating system. If $\frac{dZ(p)}{dt} \leq -\omega \|p\|$, with $\omega$ being a $K$-class function, then dynamic system under consideration is stable and zero equilibrium point (in the state space) is reached. Finding a candidate Lyapunov function in order to assess the stability from an energy dissipating perspective of a nonlinear dynamic systems such as (11) is not always a systematic procedure. The concept of Lyapunov function, however, is crucial for stability analysis when designing control laws that guarantee minimizing system energy. Converse theorem [23], ensures existence of Lyapunov function. If $f(p, k(p)) \subset C^l(K_l)$, $l$ is an integer then an appropriate Lyapunov function $Z(p)$ of class $C^1(K_p)$ exists such that

$$
\omega_1 \|p\| \leq Z(p) \leq \omega_2 \|p\|, \\
\frac{\partial Z(p)}{\partial p} f(p, k(p)) \leq -\omega_3 \|p\|, \\
\|\frac{\partial Z(p)}{\partial p}\| \leq \omega_4 \|p\|
$$

(26)

Additionally the time derivative of $Z(p)$ of (11), for $t \in [t_k, t_{k+1})$, satisfy

$$
\dot{Z} = \frac{\partial Z(p)}{\partial p} f(p, k(p)) + \frac{\partial Z(p)}{\partial p} (f(p, k(p)) - f(p, k(p_k))) \\
\leq -\omega_3 (\|p\|) + \omega_4 (\|p\|) (\|E\|)
$$
If we use triggering condition (14), and since $\omega_4$ is increasing, it holds

$$\dot{Z} \leq -\omega_3(\|p\|) + \omega_4(\|p\|_{L^\infty})$$

$$= -\mu_3(\|p\|) + \theta_3(\|p\|_{L^\infty})$$

for any $\theta \in (0, 1)$. Then we can write $\dot{Z} \leq -(1-\theta)\omega_3(\|p\|) \iff \|p\| \geq \omega_3^{-1}(\mu(\|p\|_{L^\infty})) = \mu(\|p\|_{L^\infty})$. For each sampling instant $t = t_k$, using triggering condition (14), it holds that $\dot{Z} \leq -\omega_3(\|p\|)$ and system has bounded response over the set $K_p$.

REFERENCES


