Hybrid S-Parameters for Transmission Line Networks with Linear/Nonlinear Load Terminations Subject to Arbitrary Excitations

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Abstract—We propose a generalized S-Parameter analysis for transmission lines with linear/nonlinear load terminations subject to arbitrary plane wave and port excitations. S-parameters are prevalently used to model transmission lines such as cable bundles and interconnects on printed circuit boards subject to port excitations. The conventional S-parameter approach is well suited to characterize interactions among ports. However, non-traditional port excitations associated with plane wave coupling to physical ports at transmission line terminals lead to forced as well as propagating modal waves, necessitating a modification of the standard S-parameter characterization.

In this paper, we consider external plane wave excitations as well as port (internal) sources, and propose a hybrid S-parameter matrix for characterization of the associated microwave network and systems. A key aspect of the approach is to treat the forced waves at the ports as constant voltage sources and induced propagating modal waves as additional entries (hybrid S-parameters) in the S-parameter matrix. The resulting hybrid S-matrix and voltage sources can be subsequently exported to any circuit solver such as HSPICE and Advanced Design System for the analysis of combined linear and non-linear circuit terminations at ports. The proposed method is particularly suited for susceptibility analysis of cable bundles and printed circuit boards for Electromagnetic interference evaluations. It also exploits numerical techniques for structural and circuit domain characterization and allows for circuit design optimization without a need to perform any further computational Electromagnetic analysis.

I. INTRODUCTION

S-parameters have been widely used in combining Electromagnetic (EM) analysis of transmission line networks with circuits involving linear/nonlinear loads. They have also been extensively studied for full wave extraction of parameters to characterize microwave structures ([1], [2], [3], [4]) and for integration of S-parameter networks with linear/nonlinear loads [5], [6], [7], [8], [9], [10], [11]. As opposed to representing port relations in terms of voltage and currents via Z-parameters (Impedance) or Y-parameters (Admittance), S-parameters employ modal incident and reflected electromagnetic fields to establish a physics-based mathematical relation among the ports. However, the relations among the ports are dependent on the supported modes subject to excitation. Because each mode has distinct characteristic impedance and propagation velocity, S-parameters are also referred to as modal parameters.

Port analysis is a prevalent approach for EMI/EMC and signal integrity analysis of electronic designs due to its generality and practicality. In addition to analysis of PCBs with lumped linear circuit elements, it also allows for analysis of mixed signal circuits via broad-band characterization of the entire Printed Circuit Board (PCB). Depending on whether time or frequency domain analysis is considered, various techniques can be employed to integrate the S-parameter network with circuit solvers such as HSPICE for time domain [8], [11], [7] and Harmonic Balance Method [12], [13], [14], [15] for mixed time-frequency domain characterization.

External field coupling to PCBs and transmission line networks is primarily done with Multiconductor Transmission Line Theory [16], [17], [18], [19]. However, it is not suited well for accurate analysis at high frequencies since it inherently assumes strong quasi-static analysis in its formulation. Time domain techniques such as the Finite Difference Time Domain (FDTD) method have also been used for concurrent analysis of circuits and EM structures [20]. While time domain techniques also yield accurate results, they suffer from computational inefficiencies due to the meshing of large volumes and simulation of RF-devices with high quality factors. They also run into convergence problems for circuit elements with stiff differential equations.

To tackle shortcomings of the aforementioned methods, we have recently introduced hybrid S-parameter matrix that models transmission line networks with linear/nonlinear terminations subject to both plane wave and traditional port excitations (conference papers [21] and [22]). In this paper, we extend the proposed analysis by decomposing external field excitation into forced and modal waves extracted via Generalized Pencil of
Functions and present a more practical approach to integrate forced waves with circuit analysis tools such as Advanced Design System (ADS) and HSPICE. Proposed analysis also allows for integration of evanescent modes into circuit analysis for high frequency analysis [2].

As opposed to traditional port excitations, external plane wave illumination leads to forced waves along the transmission line as well as propagating modal waves. Forced waves stem from enforcing phase matching with the incident wave along the structure walls and propagate with the wave number of the incident plane wave along the corresponding direction. Such forced waves are not affected by the loads attached to the ports. Conversely, backward and forward modal waves, originated from mismatches at port terminations, travel with corresponding eigenvalues that RF structure supports at the operating frequency. In our analysis, we consider forced waves as constant voltage sources at the ports and characterize the induced propagating modes with S-parameter matrix (hybrid S-parameters). The resulting S-matrix and voltage sources can then be exported to any circuit solver such as HSPICE and ADS and analyzed with the corresponding linear and nonlinear port terminations. Since transmission line network is solely treated in EM domain and circuit components attached to the ports are handled in the circuit domain, numerical techniques customized for each domain can be fully exploited in our analysis. This approach also allows for circuit design optimization without a need for repeated analysis of the microwave network.

Below, we first develop the theory for an arbitrary transmission line network subject to plane wave excitation. Next, we validate the proposed concept with a pair of transmission lines in free space excited by a plane wave with a current source attached to one of the terminals. Similarly, we extend our validation to a pair of microstrip lines on a PCB subject to plane wave excitation. In the final section, we discuss the proposed method and remark on future work.

II. THEORY

In this section, we consider characterization of the interactions among physical ports within a transmission line network using modal S-parameters. Subsequently, we propose a hybrid S-parameters matrix to include external plane wave excitations and proceed to describe techniques such as Generalized Pencil of Functions for the extraction of hybrid S-matrix entries. We then discuss how forced waves are treated at the ports for circuit analysis.

A. Modal S-Parameters for Coupling Among Physical Ports

Fig. 1 displays a typical mixed signal circuit board with nonuniform microstrip lines. For the characterization of such a board, we introduce the N-port S-parameter network giving (for the $k^{th}$ mode),

$$
\begin{bmatrix}
    b^k_1 \\
    \vdots \\
    b^k_N
\end{bmatrix}
= 
\begin{bmatrix}
    S^k_{1,1} & \cdots & S^k_{1,N} \\
    \vdots & \ddots & \vdots \\
    S^k_{N,1} & \cdots & S^k_{N,N}
\end{bmatrix}
\begin{bmatrix}
    a^k_1 \\
    \vdots \\
    a^k_N
\end{bmatrix}
$$

where $S^k_{ij} = \frac{k}{a_i^j}$ with all ports terminated at their corresponding reference impedance $Z_{ref_i}$, and $(a^k_i, b^k_i)$ referring to the incident and reflected waves, respectively, at the $i^{th}$ port. We must note that our analysis assume real reference impedance at the ports throughout our analysis. However, it can be readily extended to account for complex reference impedances at the ports with appropriate power relations (see [23], [24]).

We remark that even though a PCB is shown in Fig. 1, our analysis applies to any transmission line configuration as is the case with multiconductor transmission lines and coaxial cable networks. As usual, the scattering matrix assumes the following field representation due to port excitation,

$$
E(s) = \sum_k A_k e^k e^{-\gamma_k s} + \sum_k B_k e^k e^{\gamma_k s}
$$

$$
H(s) = \sum_k C_k h^k e^{-\gamma_k s} + \sum_k D_k h^k e^{\gamma_k s}
$$

where $e^k$ and $h^k$ refer to $k^{th}$ modal electric and magnetic fields, respectively, with $\gamma_k$ being the corresponding propagation constants whereas, $A_k, B_k, C_k$ and $D_k$ are the coefficients of the expansion.

The above modal fields refer to the eigensolutions of the corresponding Sturm-Liouville problem subject to Dirichlet boundary conditions on PEC surfaces

$$
\hat{n} \times e^k = 0
$$

These modal fields must also satisfy the orthogonality condition

$$
\int_S (e^p \times h^k) \cdot ds = \delta_{pk}
$$

with respect to the cross section of the propagation front along the transmission line. To relate $(e^k, h^k)$ with terminal voltages and currents, we introduce the definitions

$$
V_i^k = -\int_{C_i} \overrightarrow{E}^k \cdot d\overrightarrow{l} \\
I_i^k = -\int_{C_i} \overrightarrow{H}^k \cdot d\overrightarrow{l}
$$

The resulting S-matrix and voltage sources can then be extracted to any circuit solver such as HSPICE and ADS.
Fig. 1. (a) Typical mixed-signal PCB -(b) Port modeling of (a) where non-linear component terminals are represented with ports -(c) Circuit representation of (a) where \((V_k, I_k)\) denote voltages and currents at the \(i^{th}\) port due to the \(k^{th}\) modal field. From (2), fields to be integrated are \(E_k = A_k e^{-\gamma_k s} + B_k e^{\gamma_k s}\) and \(H_k = C_k h_k e^{-\gamma_k s} + D_k h_k e^{\gamma_k s}\).

Mathematical relation between the \(k^{th}\) incident and reflected modal amplitudes, in (1) and modal voltages and currents in (5) is given by

\[
\begin{align*}
\alpha_i^k &= \frac{V_i^k + Z_{ref_i} I_i^k}{2\sqrt{Z_{ref_i}}} \\
\beta_i^k &= \frac{V_i^k - Z_{ref_i} I_i^k}{2\sqrt{Z_{ref_i}}}
\end{align*}
\]  

(6)

where \(Z_{ref_i}\) is the reference impedance for the corresponding \(i^{th}\) port. The expression in (6) can be construed as an interface between the circuit components (expressed in terms of voltages and currents) and the RF structure treated via full-wave electric and magnetic fields. However, we must note that (6) is only applicable to modal excitations at the physical ports. Therefore, one must account for non-modal field contributions at the ports for non-conventional port excitations. Below, we exploit the relation in (6) and introduce hybrid S-matrix approach to account for non-conventional external excitations such as plane waves on transmission line networks.

B. Hybrid S-Parameters for External Plane Wave Excitation

Let us consider the case of an external plane wave impinging upon an \(N\)-port arbitrary transmission line network shown in Fig. 3. Similar to the lumped port excitations, we propose to introduce the external plane wave as a source generated from an additional \((N+1)^{th}\) port. We start our analysis by imposing Dirichlet boundary conditions along the transmission line walls (see Fig. 2) as follows,

\[
\hat{n} \times E_{total} = 0
\]

\[
\hat{n} \times (E_{inc} + E_{scat}) = 0
\]

(7)

where \(E_{inc}\) refers to electric field due to the incident plane wave in absence of the whole transmission line network, and \(E_{scat}\) is the electric field radiated by the induced currents on the transmission line conductors.

For an infinitely long transmission line, (7) implies that \(\hat{n} \times E_{scat} = -\hat{n} \times E_{inc}\) at transmission line surfaces. However, for a finite transmission line, the reflected currents at the terminals will lead to modal fields which already satisfy the boundary conditions given in (3). Therefore, the scattered fields at the transmission line surfaces satisfy the conditions

\[
\hat{n} \times E_{scat} = \hat{n} \times (-E_{inc} + E_{modal})
\]

(8)

where \(E_{modal}\) is given by (2) with the boundary conditions along the transmission line surfaces,

\[
\hat{n} \times E_{modal} = 0
\]

(9)
Referring to (8) and [25], we observe that plane wave incidence on a transmission line introduces a forced wave (having the same wave number as the incident field) in addition to the modal fields. Thus, in the case of a plane wave excitation, we introduce the representation

\[
E_{\text{total}} = E_{\text{forced}} + \sum_k A_k e^{\gamma_k s} + \sum_k B_k e^{\gamma_k s}
\]

Comparing the above expression with (2), we deduce that the difference between the plane wave and lumped port excitation is that the former induces forced waves in addition to modal fields.

To account for the modal waves coupled to the physical ports at the transmission line terminals, we treat the plane wave as an additional \((N+1)\)th port and modify the existing \(N\)-port S-matrix accordingly for each modal wave,

\[
\begin{bmatrix}
\vdots \\
 b_k^1 \\
 \vdots \\
 b_k^{N+1}
\end{bmatrix} = \begin{bmatrix}
S_{k,1}^1 & \cdots & S_{k,1}^N & H S_{k,1,N+1}^1 \\
\vdots & \ddots & \vdots & \vdots \\
S_{k,N}^1 & \cdots & S_{k,N}^N & H S_{k,N,N+1}^1 \\
S_{k,N+1}^1 & \cdots & S_{k,N+1,N}^N & H S_{k,N+1,N+1}^1
\end{bmatrix} \begin{bmatrix}
a_k^1 \\
\vdots \\
a_k^N \\
a_{k,N+1}
\end{bmatrix}
\]

As seen, the \((N+1)\)th port is characterized by the hybrid S-parameters, \(H S_{k,1,N+1}^1\), representing plane wave coupling to \(i\)th port for the \(k\)th excited mode. Fig. 3 clearly demonstrates that plane wave is included as additional port in the circuit domain and we treat forced waves as additional constant voltages at the ports. In the subsequent sections, we first describe extraction of hybrid S-parameters. Next, we explain how we integrate forced waves with circuit analysis.

C. Hybrid S-Parameters via Open Circuit Analysis

To calculate the hybrid S-parameters, we exploit the inherent relation between the incident and reflected waves, and voltage and currents at the ports given in (6). We first introduce the corresponding hybrid impedance matrix for the \((N+1)\)-port network representing voltage and current relations at the ports due to modal and plane wave excitations,

\[
\begin{bmatrix}
V_1 \\
\vdots \\
V_N \\
I_1 \\
\vdots \\
I_N \\
\alpha_{N+1}
\end{bmatrix} = \begin{bmatrix}
Z_{1,1} & \cdots & Z_{1,N} & H Z_{1,N+1} \\
\vdots & \ddots & \vdots & \vdots \\
Z_{N,1} & \cdots & Z_{N,N} & H Z_{N,N+1} \\
0 & \cdots & 0 & \alpha_{N+1}
\end{bmatrix} \begin{bmatrix}
\mathcal{I} \\
\mathcal{I}
\end{bmatrix} + \{V_{\text{oc}}^{\text{modal}}\}
\]

where \(\{V_{\text{oc}}^{\text{modal}}\} = \{HZ\}a_{N+1}\) refers to open circuit voltage at the ports due to modal fields excited by the external plane wave. Coupling to the \((N+1)\)th port is of no interest in our analysis. Therefore, it is excluded in the Z-matrix (namely, the \((N+1)\)th row of the Z-matrix is set to zero). This also helps us circumvent mismatch problems at the EMI port. The last element added to the \(\{\mathcal{I}\}\) column, \(a_{N+1}\), represents the normalized plane wave coefficient. Thus, the column, \(\{HZ\}\) can be construed as that relating the open circuit modal voltages at the ports to the incident plane wave excitation.

To associate impedance matrix entries with S-
parameters, we employ (6) to update (12) giving

\[
\sqrt{|Z_{\text{ref}}|} \{\pi\} = (13)
\]

\[
[Z](\sqrt{|Z_{\text{ref}}|})^{-1}(\{\pi\} - \{b\}) + \{V_{\text{oc}}^{\text{modal}}\}
\]

where

\[
{\pi}^T = \{a_1 \cdots a_N\} \quad \{b\}^T = \{b_1 \cdots b_N\} (14)
\]

\[
[Z] = \begin{bmatrix}
Z_{1,1} & \cdots & Z_{1,N} \\
\vdots & \ddots & \vdots \\
Z_{N,1} & \cdots & Z_{N,N}
\end{bmatrix}
\]

\[
\sqrt{|Z_{\text{ref}}|} = \begin{bmatrix}
\sqrt{Z_{\text{ref}1}} \\
\vdots \\
\sqrt{Z_{\text{ref}N}}
\end{bmatrix}
\]

in which \([Z]\) and \(\sqrt{|Z_{\text{ref}}|}\) are already known matrices.

Rearranging the terms, we find that the coefficients of the incident and reflected waves at the physical ports are given by

\[
\{b\} = \{[Z](\sqrt{|Z_{\text{ref}}|})^{-1} + \sqrt{|Z_{\text{ref}}|})^{-1} \}
\]

\[
\{[Z](\sqrt{|Z_{\text{ref}}|})^{-1} - \sqrt{|Z_{\text{ref}}|})\{\pi\} + \{[Z](\sqrt{|Z_{\text{ref}}|})^{-1} + \sqrt{|Z_{\text{ref}}|})^{-1}\{V_{\text{oc}}^{\text{modal}}\}
\]

Comparing (15) with (11) and setting \(a_{N+1} = \frac{|E_0|}{\sqrt{|Z_{\text{ref}N+1}|}}\) to normalize the incident plane wave, we readily identify that

\[
\{\chi\} = \frac{\sqrt{|Z_{\text{ref}N+1}|}}{|E_0|} \{[Z](\sqrt{|Z_{\text{ref}}|})^{-1} + \sqrt{|Z_{\text{ref}}|})^{-1}\{V_{\text{oc}}^{\text{modal}}\}
\]

where \(|E_0|\) is the magnitude of the incident plane wave and \(\{\chi\}^T = \{HS_1 \cdots HS_N\}\).

The evaluation of \([Z_{ij}]\) in (11) is done in the usual manner via open circuit analysis. However, the evaluation of \(V_{\text{oc}}^{\text{modal}}\) requires more attention. Once the open circuit modal voltages are obtained, \(\{\chi\}\) can be calculated via (16). Since the forced voltages do not depend on the terminations, they can be directly exported to the circuit solver shown in Fig. 3.

\[D. \text{Generalized Pencil of Functions for Extraction of Hybrid S-Parameters}\]

As noted above, the hybrid scattering matrix assumes the propagation of a discrete set of modes within the network. Knowledge of these modes and their associated parameters (eg. \(\gamma_k\), \(A_k\) and \(B_k\) as in (2)) is necessary for the extraction of the hybrid S-matrix entries. Generalized Pencil of Functions [26], [27] method can be employed for the extraction of these parameters. Such an analysis has been successfully employed in the literature [28], [29]. Specifically, in [28] and [29], the current induced on a microstrip line is decomposed into the bound (dominant) and higher order modes and authors employed Generalized Pencil of Functions to find the corresponding mode amplitude and propagation constants to achieve the best fit. Similarly, in [30], FDTD was used in conjunction with Generalized Pencil of Functions to extract the S-Parameters of a waveguide structure via a full wave analysis. Further, in [31], Generalized Pencil of Functions was used to extract the parameters of current induced on large scatterers represented with sum of complex exponentials.

Once the parameters of the exponential terms in (10) are attained, one can readily distinguish forced and dominant modal waves by examining the propagation constants such that the dominant modal terms appear as a pair of backward and forward travelling voltages with negligible decay/attenuation constant. A more rigorous comparison can be also made by computing the propagation constants of the transmission line network by invoking the eigenfunction representation with the appropriate boundary conditions. Since the computed eigenvalues correspond to the propagation constants, one can then extract the modal propagation constants from Generalized Pencil of Functions Method results.

\[E. \text{Integration of Forced Waves with Circuit Analysis}\]

As described above, forced and modal waves can be extracted via Generalized Pencil of Functions Method. Also, we have shown that modal waves can be combined with circuit analysis through the hybrid S-parameter matrix. Next, we describe incorporation of forced waves into the circuit analysis.

We start our analysis with the following impedance boundary condition that must be satisfied regardless of linear/nonlinear loads attached to the ports. Specifically, we have

\[
E_{\text{tan}} = Z_s H_{\text{tan}}
\]

where \(Z_s\) is measured in \(\Omega\) per square unit cell. The impedance boundary conditions for EM analysis at the ports can be translated into Ohm’s law to relate the voltage and currents at the terminals using the general form,

\[
V_{\text{total}} = f(Z_L, I_{\text{total}})
\]
where $Z_L$ is the complex linear/nonlinear load impedance at the ports. We must remark that the surface impedance $Z_s$ in (5) can be expressed in terms of the total port impedance $Z_L$ and the port dimensions. For instance, such a relation for the microstrip lines can be written as

$$Z_L = Z_s \frac{h}{w}$$ \quad (19)

where $h$ and $w$ correspond to the height of the transmission line from the ground plane and the width of the strip line, respectively.

For plane wave excitation, the total voltage at the port is expressed in terms of modal and forced waves via (10) and (5), viz.

$$V_{\text{total}} = V_{\text{forced}} + V_{\text{modal}}$$

$$V_{\text{modal}} = V_{\text{total}} - V_{\text{forced}}$$

where $V_{\text{total}}$ represents the total voltage at the terminals of any linear or nonlinear loads. As stated, $V_{\text{forced}}$ is a constant term, not associated with the loads at the ports. Thus, the forced voltage can be added to the ports as a constant source term to enforce Ohm’s law in circuit domain or equivalently the surface impedance boundary condition (see Fig. 3).

### III. Validation Studies

To demonstrate the validity of the hybrid S-parameters, we first consider a pair of transmission lines subject to concurrent plane wave and a direct port excitation. Subsequently, we consider a more complex configuration consisting of a pair of microstrip lines on a PCB illuminated by an obliquely incident plane wave.

### IV. A Pair of Transmission Lines Subject to Plane Wave Excitation

Consider the Transmission Lines (TL) shown in Fig. 4 excited by a current source at the left and terminated by a load $Z_L = 100$ located 250mm from the source. The TL is comprised of two wires of radius 0.125mm and separated by 2mm having a characteristic impedance of $Z_0 = 332.24\Omega$ [32]. We are interested in computing the voltage induced at the load when the TL is concurrently illuminated by a plane wave operating at 2 GHz (the same as the port source). This problem is therefore a typical EMI/EMC coupling analysis.

We consider the plane wave excitation $\vec{E} = \frac{E_{\text{inc}} e^{-jkr}}{\sqrt{\epsilon_r}}$ with $E_{\text{inc}} = (\hat{x}1000 + \hat{y}500 + \hat{z}1500)V/m$ and $k = k_0 \sqrt{|\epsilon_r - 1|}$. Further, we assume that the current only flows in the $\hat{y}$ direction since the wire radius is much smaller than the wavelength at the operating frequency.

We break down our analysis into two sections

1. Current source excitation
2. Plane wave excitation

Such an approach implicitly assumes linear circuit components attached to the ports. However, we must note that the proposed solution can be applied to the cases where non-linear loads are included by employing broadband S-parameter characterization.

#### A. Current Source Excitation

To compute the total voltage induced at the load, we employ S-parameter matrix defined for two ports where the current source and lumped ports are attached, respectively.

Since the current source supports quasi-TEM modes along the transmission line, one can establish a 2-Port S-Parameter network based on a quasi-TEM mode propagation. The resulting 2-Port S-Parameter network can be exported to any circuit simulator such as ADS (see Fig. 5) and connect the current source and the load at the corresponding ports.

After performing a full wave analysis (HFSS)\(^1\), we extracted $2 \times 2$ S-parameter matrix

$$S_{2 \times 2} = \begin{bmatrix} 0.909\angle 8.5 & 0.415\angle 97.7 \\ 0.415\angle 97.7 & 0.909\angle 8.5 \end{bmatrix}$$ \quad (21)

Subsequently, we exported the resulting S-matrix to ADS with the connected the current source at port 1 and the load at port 2. Using ADS (see Fig. 5) we can then find the load voltage as a function of the current source.

Table 1 shows a comparison of the full wave results with the proposed S-matrix/ADS simulation. As seen, an excellent agreement is achieved.

#### B. Plane Wave Excitation

We now proceed to include the plane wave coupling in terms of travelling wave components [25]. Referring

\(^1\)For the full wave analysis, we further use strips in place of wires by employing the standard equivalence $a = \frac{w}{2}$ where the $a$ is radius of the wire and $w$ is the width of the equivalent strip.
Voltage along the TL can be also expressed as, and backward travelling (modal) currents and the total along the wires. The remaining terms represent forward same phase as the incident field to force phase matching coupling onto infinite transmission line and have the C to Fig. 6, the V and (C V V 3

<table>
<thead>
<tr>
<th>Voltage Induced at the Z_L = 100Ω due to a Current source of 10mA</th>
</tr>
</thead>
<tbody>
<tr>
<td>I=10mA and Z_L = 100Ω</td>
</tr>
<tr>
<td>Mag(VL) (Full Wave-HFSS) 1.87</td>
</tr>
<tr>
<td>Mag(VL) (2-Port Network-ADS) 1.89</td>
</tr>
<tr>
<td>Angle(VL) (Full Wave-HFSS) 2.31</td>
</tr>
<tr>
<td>Angle(VL) (2-Port Network-ADS) 2.51</td>
</tr>
</tbody>
</table>

TABLE I

For the loads ZL1 = 250Ω, ZL2 = 800Ω, we found that

\[ V_{scat1} = (0.57 + j0.04)e^{-j24.69y} \]

\[ +(-0.37 - j1.29)e^{j42.76y} + (-0.74 - j0.61)e^{-j42.03y} \]

and for the loads ZL1 = 400Ω, ZL2 = 400Ω, we obtained

\[ V_{scat1} = (0.56 + j0.05)e^{-j24.46y} \]

\[ +(-0.77 - j1.29)e^{j42.77y} + (-0.16 - j0.37)e^{-j41.84y} \]

Considering that \( k_0 = \frac{2\pi}{\lambda} = \frac{2\pi}{0.15} = 41.88 \) and \( k_0 \sqrt{\varepsilon} = 24.14, \) it is clear that (23) and (24) are in agreement with (22). In other words, forced voltage terms did not alter with changing loads while modal waves responded to the attached loads. Based on this claim, one can establish a hybrid S-parameters network based on the quasi-TEM travelling voltages \( V_2 \) and \( V_3 \) in conjunction with the 2-port S-parameter network constructed in the previous section. Similarly, the forced voltage terms can be included in the analysis as constant voltage sources at the ports.

C. Hybrid 3-Port Quasi-TEM S-Parameter Network Construction

In this section, we proceeded to construct a 3-Port Hybrid S-Parameter network such that Port 1 and Port 2 are physical ports at the terminals of the transmission line with Port-3 representing the plane wave source leading to the quasi-TEM wave induced along the transmission lines. We first computed the open circuit modal voltages at the ports and employed (16) to compute the hybrid S-parameters. The resulting \( 3 \times 3 \) hybrid S-matrix is
Fig. 7. Circuit representation of transmission line pair subject to concurrent plane wave and current source excitation

\[
S_{3 \times 3} = \begin{bmatrix}
0.909 \angle 8.5 & 0.415 \angle 97.7 & 0.187 \angle -11.81 \\
0.415 \angle 97.7 & 0.909 \angle 8.5 & 0.190 \angle 170.0 \\
0.0 \angle 0.0 & 0.0 \angle 0.0 & 0.0 \angle 0.0
\end{bmatrix}
\]

(25)

where we set the last row to zero because only \( H_{S_{1,3}} \) and \( H_{S_{2,3}} \) are non-zero since they represent the coupling of the incident plane wave onto the physical ports at the transmission line terminals.

Next, we proceeded to employ ADS in conjunction with (25) to find the port voltages (see Fig. 7).

We performed three studies in which current source is set to zero, 5mA and 10mA respectively. Next, we compared the proposed method solution with full wave solution for the voltage induced at the load \( Z_L \) for each current source and plane wave excitation. It is clearly observed in Fig. 8 that hybrid S-parameters agree very well with full wave results.

**D. A Pair of Coupled Microstrip Lines Subject To Concurrent Plane Wave and On-Board Current Source Excitation**

We now consider the geometry in Fig. 9, displaying a pair of coupled microstrip lines residing on a RT/Duroid 5880 board with 2.2 dielectric constant and thickness of 31mils (0.7874mm).

The microstrip lines are terminated with complex impedances and an on-board current source was placed in port 1 (P1) given in Table II. Additionally, a plane wave operating at 2.5GHz (the same as the current source) also impinged on the microstrip lines.

\[
\vec{E} = E_{inc} e^{-j \vec{k} \cdot \vec{r}} \quad \text{with} \quad E_{inc} = (\hat{x}2000 + \hat{y}2000 + \hat{z}3000)V/m \quad \text{and} \quad \vec{k} = k_0 \frac{\hat{x}2000 + \hat{y}2000 + \hat{z}3000}{1.5}
\]

In our analysis, we aim to find the total voltage induced at the ports. To do so, we first extracted the standard \( 4 \times 4 \) S-parameter matrix for the given port configuration with respect to 50Ω reference impedance. Afterwards, we performed full wave analysis on the structure (with open ports) subject to only plane wave excitation. Subsequently, we conducted Generalized Pencil of Functions analysis to extract the forced voltage and propagating modes along the lines. Upon obtaining modal and forced voltages at the ports, we then em-
TABLE II
PORT TERMINATIONS IN FIG. 9

<table>
<thead>
<tr>
<th>Port</th>
<th>Impedance</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>1=50mA</td>
</tr>
<tr>
<td>P2</td>
<td>$Z_2 = 10 - j250\Omega$</td>
</tr>
<tr>
<td>P3</td>
<td>$Z_3 = 100 - j100\Omega$</td>
</tr>
<tr>
<td>P4</td>
<td>$Z_4 = 400 + j50\Omega$</td>
</tr>
</tbody>
</table>

ployed (16) to extract the hybrid S-parameters. Resulting
$5 \times 5$ hybrid S-parameter matrix is exported to ADS and
corresponding forced voltages and port terminations are
connected to the respective ports (see Fig. 10).

In this configuration, ports $P1 - P4$ correspond to
physical ports at the terminals of the coupled microstrip
lines and port $P5$ represents the plane wave coupling.
Circuit analysis was run at 2.5 GHz and performance
of the proposed method is compared with full wave
results (see Fig. 11). It is clearly observed that proposed
method results agree well with full wave results.

V. Conclusion
A novel approach for the analysis of transmission
line networks subject to a plane wave excitations was
proposed. It was shown that plane wave coupling leads
to both forced and modal waves. The former is constant
at the ports regardless of the attached load whereas, the
latter travels along the transmission lines. While modal
waves were taken into account by treating the plane wave
as an additional port, forced waves appear as constant
voltage sources at the terminals to enforce the Ohm’s law
in circuit, or equivalently, surface impedance boundary
condition in EM domain. Two validation studies were
carried out with a pair of transmission lines in free
space and also a coupled microstrip lines residing on
a PCB. It was shown that the proposed method agrees
very well with full wave results. The key advantage of
the proposed method is the treatment of EM structure in
the EM domain whereas circuit components are treated
in circuit domain. Therefore, this analysis can address a
large variety of circuit components.

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